UCRHEP-T148
July 1995

CORRELATED NEUTRINO OSCILLATIONS

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ABSTRACT

In solar neutrino oscillations, if ν_e has a significant third massive component, the allowed parameter space in Δm^2 and $\sin^2 2\theta$ for the first two components is shown to be greatly increased. This third component may be correlated to atmospheric neutrino oscillations, as shown in a specific predictive seesaw model of the 3×3 neutrino mass matrix. Possible variations to include the recent LSND results are briefly discussed.

1. Introduction

There are now three categories of data which show evidence of neutrino oscillations. (a) In experiments which detect neutrinos from the sun, there appears to be a deficit. Hence ν_e has apparently disappeared. (b) In experiments which measure the ratio ν_μ/ν_e in the atmosphere, there also appears to be a deficit. Hence a combination of ν_e and ν_μ diappearance and appearance may have also occurred. (c) The recent results of the LSND (Liquid Scintillator Neutrino Detector) experiment[1] seem to indicate that ν_e has appeared where there is originally only ν_μ.

Conventional interpretations of the above as neutrino oscillations always plot Δm^2 versus $\sin^2 2\theta$, assuming implicitly that only two neutrinos are involved in each case. As long as all mixing angles are small, this is a good approximation because the Δm^2 in each case is very different from one another. However, the atmospheric data[2] are strongly indicative of a large mixing between ν_μ and ν_e or ν_τ or both. Hence ν_e may well be composed of three mass eigenstates with a masive third component to account for all or part of the atmospheric oscillations, whereas the solar oscillations are explained by the first two components with a small Δm^2 together with a nonnegligible contribution from the massive third component. In the following it will be shown that this has the important consequence of enlarging the parameter space of Δm^2 and $\sin^2 2\theta$ for the first two components that is allowed by the present solar data.[3]

Since the LSND results are indicative of a much larger Δm^2, of order eV^2, a fourth neutrino is required if all of the data are to be explained by neutrino oscillations. This possibility will also be discussed.

2. Three-Neutrino Analysis of Solar Data

The work that I will describe in this section was done in collaboration with J. Pantaleone,[4] who has been considering neutrino oscillations of all three flavors for many years.[5] Other authors are now beginning to follow suit.[6]

Let the electron neutrino be a linear combination of three mass eigenstates:

$$\nu_e = \cos\theta\ \nu_1 - \sin\phi \sin\theta\ \nu_2 + \cos\phi \sin\theta\ \nu_3, \tag{1}$$

and assume[*]

$$\Delta m_{13}^2 \simeq \Delta m_{23}^2 \sim 10^{-2}\ \mathrm{eV}^2. \tag{2}$$

Then for a given value of θ, one may use the solar data to find the allowed region in Δm_{12}^2 and $\sin^2 2\theta_{e2} \equiv 4|U_{e2}|^2(1 - |U_{e2}|^2)$. The results for $\sin^2 2\theta = 0.35$ and 0.75 are shown below.

On the left is the allowed region for $\sin^2 2\theta = 0.35$ which differs from that of the two-neutrino analysis, *i.e.* $\theta = 0$, by only a little. However, there is already a firm indication that it has enlarged. On the right is the allowed region for $\sin^2 2\theta = 0.75$ which shows dramatically that it has greatly increased and that the adiabatic branch of the solution at around $\Delta m^2 = 10^{-4}\ \mathrm{eV}^2$ is now allowed. The dashed lines are theoretical predictions to be discussed in the next section.

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[*]Note that if m_3 were of order a few eV, reactor data would require that ν_3 overlaps very little with ν_e. See the talk by M. C. Gonzalez-Garcia, these proceedings.

3. Seesaw Structure Revealed

Recall that the well-known empirical relationship for the Cabbibo angle in terms of the ratio of the d and s quarks, *i.e.* $\sin^2\theta_C \simeq m_d/m_s$, has led to the suggestion[7] that

$$\mathcal{M}_{ds} = \begin{bmatrix} 0 & a \\ a & b \end{bmatrix}. \tag{3}$$

This simple observation has generated over the years an enormous literature on quark mass matrices. It is an especially active field of research in the past two or three years. Consider now a trivial extension of this seesaw structure and apply it to the neutrino mass matrix, namely

$$\mathcal{M}_\nu = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & a \\ 0 & a & b \end{bmatrix}, \tag{4}$$

but in the basis $\cos\theta\ \nu_e - \sin\theta\ \nu_\mu$, ν_τ, and $\cos\theta\ \nu_\mu + \sin\theta\ \nu_e$. For small a/b, the mass eigenvalues are simply 0, $-a^2/b$, and b. The usual three neutrinos are related to the mass eigenstates by

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\phi\sin\theta & \cos\phi\sin\theta \\ -\sin\theta & -\sin\phi\cos\theta & \cos\phi\cos\theta \\ 0 & \cos\phi & \sin\phi \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{5}$$

where $\sin\phi \simeq a/b \simeq \sqrt{m_2/m_3}$.

The electron neutrino is then as given by Eq. (1) and the discussion of the previous section applies. However, $\Delta m^2_{12} = m_2^2$ is now correlated with $\sin^2 2\theta_{e2}$ for a given choice of m_3 which is of course constrained by atmospheric data. In the figures above, the dashed lines represent the predictions of Eq. (4) for $m_3 \simeq 0.17$ eV (left) and 0.063 eV (right). They do indeed intersect the allowed regions.

The atmospheric neutrino oscillations are given by

$$P(\nu_\mu \to \nu_e) = \frac{1}{2}\cos^4\phi\sin^2 2\theta\left(1-\cos\frac{t\Delta m^2_{23}}{2p}\right), \tag{6}$$

$$P(\nu_\mu \to \nu_\tau) = \frac{1}{2}\sin^2 2\phi\cos^2\theta\left(1-\cos\frac{t\Delta m^2_{23}}{2p}\right). \tag{7}$$

Since the angle ϕ is small, ν_μ oscillates mainly into ν_e in this simplest realization of the seesaw ansatz. On the other hand, the $\nu_\mu - \nu_\tau$ submatrix may be rotated without affecting ν_e, in which case a better fit to the atmospheric data can be obtained.

4. A Specific Model

To obtain Eq. (4), start with ν_e, ν_μ, ν_τ, and four singlets: ν_S, N_1, N_2, N_3. Assume a discrete Z_6 symmetry [$\omega^6 = 1$] and assign

$$(\nu_e, \nu_\mu, \nu_\tau) \sim (\omega, \omega^{-2}, 1); \quad (\nu_S, N_1, N_2, N_3) \sim (\omega, 1, \omega^2, \omega^{-2}). \tag{8}$$

The Higgs sector is taken to consist of two doublets $(\Phi_1, \Phi_2) \sim (1, \omega^{-3})$ and one singlet $\chi \sim \omega$. The resulting 7×7 mass matrix is then given by

$$\mathcal{M}_7 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & m_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_4 & 0 & m_5 \\ 0 & 0 & m_3 & m_4 & M_1 & 0 & 0 \\ m_1 & m_2 & 0 & 0 & 0 & 0 & M_2 \\ 0 & 0 & 0 & m_5 & 0 & M_2 & 0 \end{bmatrix}, \tag{9}$$

where m_1 comes from $\langle\phi_2^0\rangle$, $m_{2,3}$ from $\langle\phi_1^0\rangle$, and $m_{4,5}$ from $\langle\chi\rangle$. Large $M_{1,2}$ reduce the above to a 4×4 mass matrix

$$\mathcal{M}_4 = \begin{bmatrix} 0 & 0 & 0 & m_1 m_5/M_2 \\ 0 & 0 & 0 & m_2 m_5/M_2 \\ 0 & 0 & m_3^2/M_1 & m_3 m_4/M_1 \\ m_1 m_5/M_2 & m_2 m_5/M_2 & m_3 m_4/M_1 & m_4^2/M_1 \end{bmatrix}. \tag{10}$$

Assume now that m_4^2/M_1 dominates, then

$$\begin{aligned} \mathcal{M}_3 &= \begin{pmatrix} bs^2 & bsc & as \\ bsc & bc^2 & ac \\ as & ac & 0 \end{pmatrix} \\ &= \begin{pmatrix} c & 0 & s \\ -s & 0 & c \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & a \\ 0 & a & b \end{pmatrix} \begin{pmatrix} c & -s & 0 \\ 0 & 0 & 1 \\ s & c & 0 \end{pmatrix}, \end{aligned} \tag{11}$$

where $b = (m_1^2 + m_2^2) m_5^2 M_1 / m_4^2 M_2^2$, $a = m_3 m_5 \sqrt{m_1^2 + m_2^2} / m_4 M_2$, $s = m_1 / \sqrt{m_1^2 + m_2^2}$, $c = m_2 / \sqrt{m_1^2 + m_2^2}$. The desired seesaw structure is thus obtained.

5. Addition of a Fourth Neutrino

Since Eq. (10) contains a fourth neutrino which couples to both ν_e and ν_μ, it may be considered as a candidate for explaining the recent LSND results.[1] However, the required mass and mixing of this singlet neutrino with ν_e are then too large to be consistent with the nucleosynthesis bound on the number of light neutrinos.[8] To avoid this problem, the most natural thing to do is to use the singlet neutrino to explain the solar data in the matter-enhanced small-angle nonadiabatic solution, as has been pointed out by many authors.[9] In that case, ν_μ and ν_τ may be assumed to have masses of a few eV, but a small enough mass difference and large enough mixing to account for the atmospheric data. A small mixing between ν_e and ν_μ may then be invoked to explain the LSND results. A recently proposed model[10] uses a discrete Z_5 symmetry and the seesaw reduction of a 7×7 mass matrix to obtain four approximate light neutrino mass eigenstates $\cos\theta\ \nu_e - \sin\theta\ \nu_S$, $\cos\theta\ \nu_S + \sin\theta\ \nu_e$, $(\nu_\mu + \nu_\tau)/\sqrt{2}$, and $(\nu_\mu - \nu_\tau)/\sqrt{2}$, with eigenvalues 0, m_1, m_2, and $-m_2$ respectively. In addition, mixing occurs between ν_e and ν_μ, as well as between ν_S and ν_τ. Note that ν_μ and ν_τ are pseudo-Dirac partners, hence $\sin^2 2\theta = 1$ is required for atmospheric neutrino oscillations.

To accommodate a fourth neutrino in the present context, a possible variation is to double Eq. (3) and consider the 4×4 mass matrix

$$\mathcal{M}'_\nu = \begin{bmatrix} 0 & a & 0 & 0 \\ a & b & 0 & 0 \\ 0 & 0 & 0 & c \\ 0 & 0 & c & d \end{bmatrix} \tag{12}$$

in the basis $\cos\theta\ \nu_e - \sin\theta\ \nu_\mu$, ν_S, $\cos\theta\ \nu_\mu + \sin\theta\ \nu_e$, and ν_τ. Solar neutrino oscillations are as given before, but now the second mass eigenstate is mostly inert and there is no phenomenological constraint on the ratio a/b as in the case of Eq. (4). Atmospheric neutrino oscillations are mostly between ν_μ and ν_e, whereas the LSND results are explained by the fact that both ν_e and ν_μ mix with ν_τ. However, because of the seesaw ansatz, the latter is correlated with the former. Numerically, they are indeed consistent with both sets of data, although the value of Δm^2 in the LSND case is required to be less than about 3 eV^2.

6. Conclusions

As more neutrino experiments accummulate more data, there are two important messages for phenomenologists and model builders. First, the naive assumption that each case of neutrino oscillations is to be interpreted as between only two mass eigenstates must be abandoned. Atmospheric data tell us that a large mixing angle exists between ν_μ and ν_e or ν_τ or both. In this talk it has been shown that if ν_e has a significant third massive component, the analysis of solar data allows a much larger parameter space in Δm^2 and $\sin^2 2\theta$ for the first two components.

Second, the structure of the neutrino mass matrix is beginning to reveal itself. It is time to look for possible empirical relationships such as the well-known $\sin^2\theta_C \simeq m_d/m_s$ for quarks which may give us a glimpse of the underlying theory of the origin of masses. In this talk a first attempt, *i.e* Eqs. (4) and (12), has been noted.

7. Acknowledgements

I thank Prof. J. W. F. Valle and the other organizers of the International Workshop on Elementary Particle Physics: Present and Future (Valencia 95) for their great hospitality and a very stimulating program. This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837.

8. References

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